



MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

- *This Question Paper consists of three sections A, B and C.*
- *Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.*
- *Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.*
- *Section B: Internal choice has been provided in one question of two marks and one question of four marks.*
- *Section C: Internal choice has been provided in one question of two marks and one question of four marks.*
- *All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.*
- *The intended marks for questions or parts of questions are given in brackets [].*
- *Mathematical tables and graph papers are provided.*

SECTION A - 65 MARKS

Question - 1

(i) If $[P + Q]^T = \begin{bmatrix} 7 & 9 \\ 0 & 11 \end{bmatrix}$ if $P = \begin{bmatrix} 5 & -1 \\ 6 & 0 \end{bmatrix}$

(a) $Q = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

(c) $Q = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

(d) None of these

(ii) If $\int |x| dx = kx|x| + C, x \neq 0$, then find the value of k.

(a) $k = \frac{1}{2}$

(b) $k = \frac{1}{2}$

(c) $k = \frac{1}{2}$

(d) None of these

(iii) The value of $\cot \left(\cos^{-1} \frac{5}{3} + \tan^{-1} \frac{2}{3} \right)$ is

- (a) $\frac{6}{17}$
- (b) $\frac{3}{17}$
- (c) $\frac{4}{17}$
- (d) $\frac{5}{17}$

(iv) $\frac{dy}{dx} = \frac{x^n - y^n}{xy + y^2}$ is

- (a) $n=3$
- (b) $n=5$
- (c) $n=2$
- (d) $n=4$

(v) If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{10}$, then $P(B/A)$ is equal to

- (a) $\frac{3}{5}$
- (b) $\frac{3}{10}$
- (c) $\frac{1}{2}$
- (d) None of these

(vi) If $A = \begin{bmatrix} x & 2 \\ 3 & x \end{bmatrix}$ and $|A| = 625$, find the real value(s) of x .

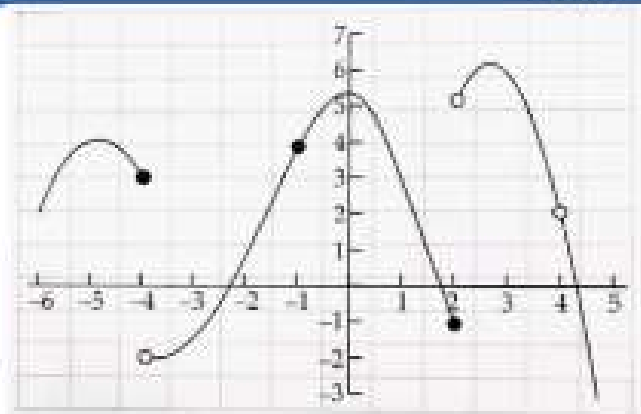
- (a) $x=3, -3$
- (b) $x=2, 4$
- (c) $x=3, 6$
- (d) $x=2, 2$

(vii) Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$.

- (a) $\sin 2x$
- (b) $\tan x$
- (c) $\cos x$
- (d) $\cos 3x$

(viii) Show below is the graph of a function, $f(x)$.

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- (a) Point of discontinuity are (1, 3, -6)
 (b) Point of discontinuity are (-4, 2, 4)
 (c) Point of discontinuity are (0, 2, 4)
 (d) Point of discontinuity are (6, -4, 4)

(ix) Let $f: R \rightarrow R$ be a function defined by $f(x) = |x|$. Then, f is

- (a) Both one-one and onto
 (b) One-one but not onto
 (c) Onto but not one-one
 (d) Neither one-one nor onto

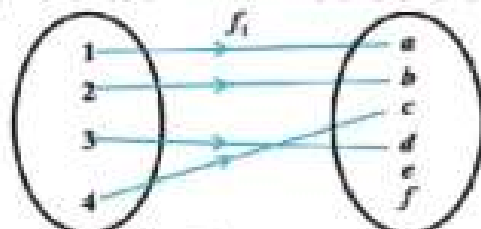
(x) Let A and B be two matrices such that both AB and BA are defined.

Assertion (A) : $(A + B)(A - B) = A^2 - B^2$.

Reason (R) : $(A + B)^2 = A^2 + 2AB + B^2$.

- (a) Both A and R are correct and R is the correct explanation of A.
 (b) Both A and R are correct but R is not the correct explanation of A.
 (c) A is correct but R is not correct.
 (d) A is not correct but R is correct.

(xi) The function $f: X \rightarrow Y$ is one-one. Give reason.



(xii) If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then find A^3 .

(xiii) A relation R on the set $A = \{1, 2, 3, \dots, 14\}$ is defined as $R = \{(x, y) : 3x = y\}$. then R is

- (a) Reflexive

- (b) Reflexive but not symmetric
- (c) Symmetric but not transitive
- (d) Neither reflexive, nor symmetric, nor transitive

(xiv) Two cards are drawn at random and one by one without replacement from a well shuffled pack of 52 playing cards, find the probability that one card is red and the other is black.

(xv) Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

- (a) $\frac{7}{9}$
- (b) $\frac{2}{11}$
- (c) $\frac{1}{9}$
- (d) $\frac{6}{11}$

Question -2

(i) Find $\frac{dy}{dx}$, if $x = at^2$ and $y = 2at$.

OR

(ii) The volume of a cube is increasing at the rate of $9 \text{ cm}^3/\text{s}$. How fast is the surface area increasing when the length of an edge is 10 cm?

Question -3

Evaluate: $\int_e^{e^4} \frac{dx}{x \log x}$

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Question -4

Find the point on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

Question -5

- (i) Evaluate the integral: $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$.
- (ii) Evaluate the integral: $\int e^{\sqrt{x}} dx$.

Question -6

Find the range of the function: $f(x) = 3 \sin x - 4 \cos x$.

Question -7

If $\sin^{-1} x + \tan^{-1} x = \frac{\pi}{2}$ prove that $2x^2 + 1 = \sqrt{5}$.

Question -8

Evaluate: $\int \frac{x^2}{x^2+3x-3} dx$.

Question -9

- (i) The value of a and b , for which $f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$ is differentiable at $x=1$.
- (ii) If $y = a \cos(\log x) + \sin(\log x)$, prove that $x^2 y_2 + x y_1 + y = 0$.

Question -10

Two teams (A and B) toss coins. In each round, all 4 players (2 per team) toss. If team A has both heads and team B doesn't, A wins instantly. Probability per round for A to win is $p = 9/64$. They play rounds until A wins.

- (a) Probability A wins in one round?
 (b) Probability the game lasts exactly 3 rounds?

OR

A retail chain has:

- Downtown stores: 40% of customers, satisfaction rate 80%
- Mall stores: 35% of customers, satisfaction rate 70%
- Outlet stores: 25% of customers, satisfaction rate 90%

A highly satisfied customer is chosen.

- (a) Represent as probability events.
 (b) Find probability they shopped at an Outlet store.
 (c) Which store type is most likely given high satisfaction?

9

Question -11

Solve the following system of equations, using matrix method:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} + \frac{20}{z} = 2; x, y, z \neq 0.$$

Question -12

- (i) Find the particular solution of the differential equation: $(1+x^2) \frac{dy}{dx} = (e^{\tan^{-1} x} - y)$, given that $y = 1$ where $x = 0$.
- (ii) Evaluate: $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$.

Question -13

- (i) A square tank of capacity 250 cubic metres has to be dug out. The cost of land is Rs.50 per sq. m. The cost of digging increases with the depth and for the whole tank it is $400xh^2$ where h metres is the depth of the tank. What should be the dimensions of the tank so that the cost would be minimum?
- (ii) Find the maximum surface area of a circular cylinder that can be inscribed in a sphere of radius R .

Question -14

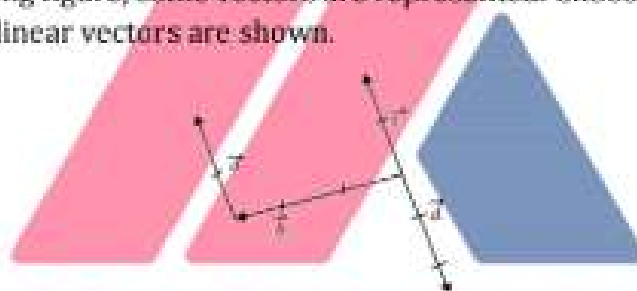
There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean of X .

Section B 15 MARKS

Question -15

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) In the following figure, some vectors are represented. Choose the correct option where all collinear vectors are shown.



- (a) $\vec{a}, \vec{c}$ and $\vec{d}$ (b) $\vec{a}$ and $\vec{b}$ (c) $\vec{c}$ and $\vec{d}$ (d) $\vec{a}$ and $\vec{b}$
- (ii) Write the vector equation of the line passing through the point $(5, 2, -4)$ and parallel to the vector $3\vec{i} + 2\vec{j} - 8\vec{k}$.
- (iii) Find the area of a parallelogram whose adjacent sides with diagonals are given by the vectors $\vec{a} = 3\vec{i} + \vec{j} + 4\vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$.
- (iv) Angle between $\vec{i} + \vec{j} + \vec{k}$ and $\vec{i} - \vec{j} + \vec{k}$ is
 (a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(-\frac{1}{3}\right)$ (c) $\sin^{-1}\left(\frac{1}{3}\right)$ (d) none of these
- (v) Find $\vec{a} \cdot (\vec{b} \times \vec{c})$, if $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} + \vec{k}$ and $\vec{c} = 3\vec{i} + \vec{j} + 2\vec{k}$.

Question -16

- (a) The projection of a vector on the coordinate axes are 6, -3, 2. Find its length and direction cosines.

OR

- (b) If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$.

Question -17

- (a) Vectors B and C of triangle ABC lie along the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4}$. Find the area of the triangle given that A has coordinates (1, -1, 2) and line segment BC has length 5.

OR

- (b) Find the shortest distance between the lines $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$; $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

Question -18

Sketch and shade the area of the region lying in first quadrant and bounded by $y = 9x^2$, $x = 0$, $y = 1$ and $y = 4$. find the area of the shaded region.

Section C 15 MARKS

Question -19

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) The A company produces a commodity with Rs. 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of Rs. 8 per unit. Find the break-even point.
- (ii) The slope of the regression line of Y on X is also called the:
 - (a) Correlation coefficient of X on Y
 - (b) Correlation coefficient of Y on X
 - (c) Regression coefficient of X on Y
 - (d) Regression coefficient of Y on X
- (iii) If the regression coefficient for the variable x and y are $b_{yx} = 1$ and $b_{xy} = \frac{1}{2}$ then find the angles between the regression lines.
- (iv) The marginal revenue function of a commodity is given by $MR = 11 - 3x + 4x^2$. Find the revenue function.
- (v) The cost of over haul of an engine is Rs 10,000. The operating cost per hour is at the rate of $2x - 240$ where the engine has run x km. Find out the total cost if the engine run for 300 hours after overhaul.

Question 20.

For the lines of regression $4x - 2y = 4$ and $2x - 3y + 6 = 0$, find the mean of 'x' and the mean of 'y'.

OR

The selling price of a commodity is fixed at ₹60 and its cost function is $C(x) = 35x + 250$
 (i) Determine the profit function.

(ii) Find the break even points.

Question 21.

Find the equation of the regression line of y on x , if the observations (x, y) are as follows: $(1, 4)$, $(2, 8)$, $(3, 2)$, $(4, 12)$, $(5, 10)$, $(6, 14)$, $(7, 16)$, $(8, 6)$, $(9, 18)$.

Also, find the estimated value of y when $x=14$

Question 22.

(i) Manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs.30 per trunk on the first type of trunk and Rs.25 per trunk on the second type. Formulate a linear programming problem to find out how many trunks of each type he must make each day to maximize his profit.

OR

(ii) A new cereal, formed of a mixture of bran and rice, contains at least 88 grams of protein and at least 36 milligram of iron. Knowing that bran contains 80 gram of protein and 40 milligram of iron per kilogram, and that rice contains 100 gram of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing a kilogram of this new cereal if bran costs Rs.28 per kilogram and rice costs Rs.25 per kilogram.

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